Record-Edge Selection and Dyadic Bohr Capture of Off-Line Spectral Forcing

Best-of-Breed v1.6 — defect–compensator closure, Euler negative control, and Regime-I survey program

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August 14, 2026

STATUS (August 28, 2026): WORKING DRAFT — RESEARCH IN PROGRESS. This document is the current working paper of the Record-Edge research program, posted for transparency of the research trail. Open questions are labeled as such throughout; no proof of RH is claimed. **Do not cite as established results.** The published, refereed output of this program to date is: M. J. Goss Jr., *The One-Direction Obstruction*, Zenodo, DOI `10.5281/zenodo.22150483`. *Provenance:* this file carries the author's final v1.6 text (August 14, 2026, 20:10 revision), typeset from the author's LaTeX source of the same date. The version history v1.2–v1.6 is published as a Zenodo version chain, concept DOI `10.5281/zenodo.22150690`.

Abstract

Version 1.6 preserves the complete v1.5 record-edge and intrinsic-source architecture. The five record-edge gates remain Gaussian persistence (GP), lower-block separation (LB), quantitative Bohr relative density (QB), heat-flow centered-prime mean square (CPM), and common selection (SI); none is declared solved. The v1.5 positive source

$$M_\psi = \sum_{n\geq 2} \frac{\Lambda(n)}{n}\,\delta_{\log n} = (\partial_v + 1)(\widetilde{F} + 1), \qquad \widetilde{F}(v) = e^{-v}\big(\psi(e^v) - e^v\big),$$

and its injective Gaussian trajectory are retained unchanged, as are the A6 gauge obstruction and the open arithmetic-separator and zero–source bridge placeholders.

The new v1.6 contribution is a structural audit layer based on *defect versus compensator closure*. For a normed observer space Y, a defect $b \in Y$, and a linear compensator map $B : C \to Y$, the decisive quantity is

$$\delta_Y(b) := \operatorname{dist}\Big(b, \overline{B(C)}^{\,Y}\Big).$$

A genuine bore exists exactly when $\delta_Y(b) > 0$, equivalently when a continuous functional annihilates the full closed compensator range while detecting the defect. This separates exact noncompensation from the stronger and load-bearing notion of non-approximability.

A certified negative control is added. For every compact $I = [\sigma_0, \sigma_1] \subset (0, \infty)$, the prime-ray Euler family

$$g_p(s) = \frac{1}{p^{s+1} - 1}$$

has dense real span in $C(I)$. Hence the algebraic range is proper but its closure is all of $C(I)$, $\ker B^* = \{0\}$, and no continuous Euler bore exists in the uniform $C(I)$ topology. This result is used as a calibration test for every finite-dimensional numerical approximation: a positive finite residual is not evidence of a bore if it erodes as the compensator model is enriched.

That calibration has now been executed in three distinct numerical regimes. Double-precision minimax approximation exhibits effective-rank saturation and an apparent residual floor; a 50-decimal-digit rerun drives the $N = 20$ apparent sup residual from order 10^{-5} to order 10^{-13} at the cost of coefficient norms of order 10^{16}; and Tikhonov regularization produces a stable-looking positive residual plateau even though the true closure gap is rigorously zero. These runs certify no new theorem, but they provide a paper-internal numerical fingerprint for three false-gap mechanisms: finite-precision rank collapse, precision-retreat with coefficient blow-up, and regularization bias.

The lower-block search is then placed in a conservative Regime I. Write

$$A(v) := \widetilde{F}(v) + 1 = e^{-v}\psi(e^v), \qquad r(v) := A(v) - 1 = \widetilde{F}(v).$$

The classical unconditional PNT error estimate gives $r \in L^2([0,\infty))$, whereas $A(v) \to 1$ and therefore $A \notin L^2([0,\infty))$. Accordingly the signed lower-block observer is r and the first certified Hilbert arena is

$$Y_0 = L^2([0,\infty)).$$

For exponential weights $Y_a = L^2(e^{av}dv)$, $a > 0$, v1.6 makes no unproved membership claim: $a = 0$ is the current *certified* endpoint of the exponential-weight family, not a theorem that $r \notin Y_a$ for every $a > 0$.

The Regime-I decomposition is now proved: $r = b_{\rho_0} + R_{\rho_0} + a_{\text{fixed}}$ with all terms in $L^2([0,\infty))$, and the freedom audit classifies the remainder as fixed—no genuine compensator range exists in this decomposition, so the operator-theoretic closure question is inapplicable. The correct continuation is a localized signal–background separation on finite-height record edges, which returns the problem to the five-gate architecture GP/LB/QB/CPM/SI with structural justification from the closure audit. KANDy or any other numerical search is explicitly exploratory: it may locate stable phase boundaries or candidate dual witnesses, but no finite-dimensional separation is promoted as proof. No proof of RH is claimed.

1 Status, provenance, and audit trail

This document supersedes v1.5 as the current working paper while preserving v1.2–v1.5 unchanged as historical predecessors. The ambient problem is Riemann's 1859 hypothesis on the nontrivial zeros of $\zeta(s)$ [1]; standard zeta-function conventions follow Titchmarsh–Heath-Brown [2].[1] The v1.2 contour proof has separately passed Gate A; in particular, the cancellation of the apparent $s = 0$ pole in

$$Q(s) = -\frac{1}{s} - \frac{1}{s-1} + \frac{1}{2}\log\pi - \frac{1}{2}\psi(s/2)$$

through $\psi(s/2) = -2/s + O(1)$ is retained without alteration.

[1]The rigorous finite-height verification of Platt and Trudgian through height $3 \cdot 10^{12}$ [11] is contextual only. No finite computation is used to replace any analytic gate or closure placeholder in this paper.

v1.2 mechanisms retained

Mechanism	Status in v1.6
Canonical Gaussian weight W_α	Retained; exact ξ-coupled contour provenance.
Canonical identity $\mathcal{S}^{\mathrm{G}}_\alpha = \mathcal{P}^{\circ}_\alpha + \mathcal{R}^{\mathrm{can}}_\alpha$	Retained; $\mathcal{R}^{\mathrm{can}}_\alpha = O_\alpha(t^{-1/2})$.
Functional-equation quartet geometry	Retained exactly.
Finite-height record edge $\beta(H)$	Retained; no global rightmost zero is assumed.
Gaussian high-tail bound	Retained.
Bohr nonvanishing and dyadic propagation	Retained qualitatively.
Second-moment contradiction architecture	Retained.

StrandLoop P01 audit ledger

The first ten-strand packet was passed through five independent loop arms (control, implication, construction, invariant, completion). Version 1.4 retains the P01 dispositions below and adds a separate P02–P03 ledger; only conclusions surviving direct mathematical checking are promoted.

Strand	Disposition	retained action
S1	Closed	Exact erfc formula for K_α promoted to theorem.
S2	Closed	Exact Stieltjes and PNT-error integral formulas for $\mathcal{P}^{\circ}_\alpha$ promoted.
S3	Closed	Exact Gaussian convolution identity promoted.
S4	Corrected	Heat-semigroup notation retained by convolution definition; unconditional global tempered-distribution Fourier language is not assumed.
S5	Corrected	Replace $r_H^2 > H^2/2 + o(H^2)$ heuristic by a log-sensitive fresh-record obstruction.
S6	Corrected	Replace the ratio-only $\sqrt{2}$ threshold by the surplus $H^2 - 2h^2$ plus explicit amplitude errors.
S7	Corrected	Retain $\log 4$ in the exact lower-block criterion; remove the false equivalence without an $\eta_H H^2$ hypothesis.
S8	Sharpened target	Simultaneous approximation alone does not certify the relative-density gap L_H; quantitative Bohr recurrence remains open.
S9	Closed	Restore the Jacobian e^u and obtain bounded-width exponent equivalence.
S10	Sharpened target	Average energy on a superlevel set does not by itself produce a fixed-length interval; an interior-thickness/selection interface is required.

StrandLoop P02–P03 audit ledger

Pass 2 iterated the P01 synthesis and exposed two useful structures: an exact local-energy defect for the forced prime lower bound, and a second half-normalization that places the arithmetic source unconditionally in $\mathcal{S}'$. Pass 3 then stress-tested those structures through five further arms. The following changes are promoted.

Audit item	v1.4 disposition
Normalized source $\widetilde{F}$	Promoted. The elementary bound $\lvert\widetilde{F}(v)\rvert \ll 1+\lvert v\rvert$ gives unconditional temperedness.
Normalized prime field $\widetilde{\mathcal{P}_\alpha^\circ}$	Promoted. It is an exact convolution $k_\alpha * \widetilde{F}$ with Fourier multiplier m_α.
Backward heat flow	Restricted. Division by m_α is valid on the exact Gaussian range only; m_α^{-1} is not a global multiplier on $\mathcal{S}'$.
Analytic regularization	Promoted as a structural corollary: $\widetilde{\mathcal{P}_\alpha^\circ}$ is smooth of polynomial growth and extends entire with Gaussian vertical growth.
Local energy defect	Promoted after repair. The defect is defined directly from the prime norm, so CPM implies nonpositive defect tautologically.
P03 S16 implication as originally written	Rejected as standalone syntax: without the defect definition, the displayed prime-norm bound does not imply $\mathcal{D}_H \le 0$. The repaired definition is included in Section 20.
Global inverse-operator language	Not promoted. No continuity or global well-posedness of backward heat flow is claimed.

v1.5 A6/source-invariant and instrument audit ledger

The August 13 continuation produced two mathematically distinct conclusions and two experimental cautions. Only the conclusions that survive direct checking are promoted below; StrandLooper agreement is provenance, not proof.

Audit item	v1.5 disposition
A6 diagonal gauge obstruction	Promoted as an abstract identifiability theorem: Q-factored observers cannot recover a component that varies along a nontrivial diagonal gauge orbit.
Positive atomic source M_ψ	Promoted. It is a positive tempered Radon measure and is linked to $\widetilde{F}$ by the exact distributional identity $M_\psi = (\partial_v + 1)(\widetilde{F} + 1)$.
Gaussian source injectivity	Promoted. The map $M \mapsto h_\alpha * M$ is injective on $\mathcal{S}'(\mathbb{R})$; the actual source M_ψ lies in that space.
Intrinsic source invariant $\mathfrak{U}_\alpha(Q)$	Promoted. It is the singleton $\{\widehat{M_\psi}\}$ and is independent of α.
Positive-definite source criterion	Promoted as standard harmonic analysis: $M_\psi \geq 0$ tempered implies $\widehat{M_\psi}$ is positive definite. This is a source-cone characterization, not an RH discriminator by itself.
Inverse heat flow	Retained as unstable/restricted. Uniqueness of the source does not imply a continuous global inverse heat operator.
Arithmetic separator Σ	Open. It must distinguish the actual arithmetic source from generic positive sources and must enter an exact off-line-zero incompatibility.
Round-3 semantic-type loss	Recorded as an instrument error. Preserving a symbol without preserving its mathematical type is not accepted as semantic preservation.
Final-round source-ID isolation	Recorded as an orchestration error. Model-generated labels such as "F1"–"F5" are not execution isolation unless the engine supplies distinct real source identifiers.

v1.6 defect–compensator and Regime-I audit ledger

The August 14 continuation is promoted only where the mathematics is definition-complete. The general quotient geometry and the Euler $C(I)$ density theorem are proved below. The Regime-I L^2 membership statement is proved from the unconditional PNT error estimate. The exact zero-side compensator map in Regime I remains an open definition/closure gate.

Audit item	v1.6 disposition
Defect–compensator quotient geometry	Promoted. Positive bore margin means positive distance from the *closed* compensator range, equivalently a continuous dual annihilator that detects the defect.
Prime-ray Euler closure in $C(I)$	Promoted as a negative-control theorem: the prime-ray range is proper but dense, so $\ker B^* = \{0\}$ and every continuous bore margin is zero in this topology.
Canonical primitive $A = \widetilde{F} + 1$	Retained from Proposition 6.3; no arbitrary inversion constant is introduced when A is defined directly from $\widetilde{F}$.
Signed lower block $r = \widetilde{F}$	Promoted as the Regime-I observer. The unconditional PNT error gives $r \in L^2([0,\infty))$, while $A \notin L^2([0,\infty))$.
Exponential weights e^{av}, $a > 0$	**Not promoted as admissible or inadmissible.** The frozen paper does not prove weighted membership for any fixed $a > 0$; the PNT upper bound alone cannot prove nonmembership. Thus $a = 0$ is the current certified endpoint, not an exclusion theorem.
Regime-I defect b_{ρ_0} and compensator map B_0	**OPEN/UNDERDEFINED** until an exact lower-block explicit-formula decomposition is written in $L^2([0,\infty))$ with the full zero orbit and all background terms.
Internal wall survey / KANDy	Active as research instrumentation only. The Euler theorem is the mandatory negative control; finite residuals $J_N > 0$ are never promoted to positive closure distance.

Logical status. The contour identity, kernel closure, PNT-error reduction, convolution identity, quartet decomposition, finite-height decomposition, and finite-height capture theorem are proved. The asymptotic existence of an admissible sequence remains conditional. The revised targets in Sections 16–24 are research gates, not imported conclusions from the strand generator.

2 Bohr nonvanishing produces recurrent thick intervals

Definition 2.1. *Following Bohr's classical theory [5], a bounded continuous function $G : \mathbb{R} \to \mathbb{C}$ is* Bohr uniformly almost-periodic *if for every $\varepsilon > 0$ the set*

$$\mathcal{T}_\varepsilon(G) := \{\tau : \sup_{u\in\mathbb{R}} |G(u+\tau) - G(u)| < \varepsilon\}$$

is relatively dense.

Lemma 2.2 (Bohr thick-superlevel recurrence)**.** *Let G be nonzero and Bohr uniformly almost-periodic. Then there exist $q > 0$, $\ell > 0$, and $L > 0$ such that every interval of length $L + 2\ell$ contains a subinterval J of length 2ℓ on which*

$$|G(u)| \geq q \qquad (u \in J).$$

Consequently the superlevel set $E_q := \{u \geq 0 : |G(u)| \geq q\}$ has positive lower density.

Proof. Choose u_0 with $|G(u_0)| = a > 0$. By continuity choose $\ell > 0$ such that $|G(u)| \geq 3a/4$ for $|u - u_0| \leq \ell$. Put $\varepsilon = a/4$ and $q = a/2$. Relative density supplies L such that every interval of length L contains a translation τ with $\|G(\cdot + \tau) - G\|_\infty < a/4$. On the translated 2ℓ-interval we then have $|G| \geq a/2$. Packing disjoint blocks gives positive lower density. □

3 Bohr–dyadic multiplicative propagation

Lemma 3.1 (Dyadic localization)**.** *If $E \subset [0, \infty)$ is measurable with positive lower density, then there exist $c_u > 0$ and infinitely many integers n such that*

$$\operatorname{meas}\big(E \cap [n \log 2, (n+1) \log 2]\big) \geq c_u.$$

Proof. Otherwise, for every m all sufficiently large dyadic log-cells would contain less than $1/m$ of E. Summing cells and dividing by total length would force lower density zero. □

Theorem 3.2 (Bohr–Dyadic Multiplicative Propagation)**.** *Let G be nonzero and Bohr uniformly almost-periodic. Then there exist $q, c_t > 0$ and infinitely many dyadic scales $T = 2^n$ such that*

$$\operatorname{meas}\{t \in [T, 2T] : |G(\log t)| \geq q\} \geq c_t T.$$

Moreover, for every $\beta > 0$,

$$\int_T^{2T} t^{2\beta-1} |G(\log t)|^2 \, dt \geq C(G, \beta) T^{2\beta}$$

along the same sequence.

Proof. Apply Lemmas 2.2 and 3.1 to the superlevel set. Under $t = e^u$, positive log-measure inside a dyadic log-cell becomes positive linear measure in $[T, 2T]$; on that interval $t^{2\beta-1} \asymp_\beta T^{2\beta-1}$. □

4 Canonical completed-zeta transform

Let

$$\xi(s) = \frac{1}{2} s(s-1) \pi^{-s/2} \Gamma(s/2) \zeta(s), \qquad \xi(s) = \xi(1-s),$$

and fix $\alpha > 0$. Define

$$W_\alpha(s) := e^{\alpha(s-1/2)^2}. \tag{1}$$

Then

$$W_\alpha(1-s) = W_\alpha(s), \qquad W_\alpha(\overline{s}) = \overline{W_\alpha(s)},$$

and on $0 \leq \sigma \leq 1$,

$$|W_\alpha(\sigma + i\gamma)| = e^{\alpha((\sigma-1/2)^2 - \gamma^2)} \leq e^{\alpha/4} e^{-\alpha\gamma^2}. \tag{2}$$

For $c > 1$ define

$$\mathfrak{X}_\alpha(t) := \frac{1}{2\pi i} \int_{(c)} -\frac{\xi'}{\xi}(s) \frac{W_\alpha(s)}{s} t^{s-1/2} \, ds, \qquad t > 1. \tag{3}$$

Definition 4.1 (Canonical Gaussian zero field)**.** *Let $\mathcal{Z}$ be the multiset of nontrivial zeros, counted with multiplicity. Define*

$$\mathcal{S}_\alpha^{\mathrm{G}}(t) := -\sum_{\rho \in \mathcal{Z}} \frac{W_\alpha(\rho)}{\rho} \, t^{\rho-1/2}. \tag{4}$$

Lemma 4.2 (Gaussian coefficient tail)**.** *There is $K_\alpha > 0$ such that for $H \geq 2$,*

$$\sum_{|\gamma|>H} \left| \frac{W_\alpha(\rho)}{\rho} \right| \leq K_\alpha e^{-\alpha H^2/2}. \tag{5}$$

Proof. Use (2), $|\rho|^{-1} \ll |\gamma|^{-1}$ for $|\gamma| \geq 2$, and the standard unit-height zero count $N(Y+1) - N(Y) = O(\log(Y+2))$. □

4.1 Exact prime–zero coupling

For $\Re s > 1$, using the standard logarithmic-derivative expansion [2, 4],

$$-\frac{\xi'}{\xi}(s) = \sum_{n\geq 2} \frac{\Lambda(n)}{n^s} + Q(s), \tag{6}$$

where

$$Q(s) := -\frac{1}{s} - \frac{1}{s-1} + \frac{1}{2}\log\pi - \frac{1}{2}\psi(s/2). \tag{7}$$

Define

$$K_\alpha(x) := \frac{1}{2\pi i}\int_{(c)} \frac{W_\alpha(s)}{s} x^s\, ds \qquad (c > 1) \tag{8}$$

and

$$\mathcal{P}_\alpha^\circ(t) := t^{-1/2}\sum_{n\geq 2}\Lambda(n)K_\alpha(t/n) - e^{\alpha/4}t^{1/2}. \tag{9}$$

Theorem 4.3 (Canonical Gaussian explicit-formula identity)**.** *For fixed $\alpha > 0$ and $t > 1$,*

$$\boxed{\mathcal{S}_\alpha^{\mathrm{G}}(t) = \mathcal{P}_\alpha^\circ(t) + \mathcal{R}_\alpha^{\mathrm{can}}(t)}, \tag{10}$$

where

$$\mathcal{R}_\alpha^{\mathrm{can}}(t) = O_\alpha(t^{-1/2}). \tag{11}$$

More precisely, shifting to $\Re s = -1$ gives two vertical integrals of size $O_\alpha(t^{-3/2})$ plus the $s = 0$ kernel-pole term of size $O_\alpha(t^{-1/2})$.

Proof. Shift (3) from $\Re s = c > 1$ to $\Re s = -1$ along rectangles avoiding zero ordinates. Gaussian vertical decay removes the horizontal sides. The residues of $-\xi'/\xi$ are the nontrivial zeros with multiplicity, giving (4); the additional $1/s$ kernel gives the $s = 0$ term.

On the original line insert (6). The von Mangoldt series gives (9) except for the Q-integral. Shift the latter to $\Re s = -1$. Its apparent pole at $s = 0$ cancels because

$$\psi(s/2) = -\frac{2}{s} + O(1),$$

while the crossed pole at $s = 1$ contributes $-W_\alpha(1)t^{1/2} = -e^{\alpha/4}t^{1/2}$. On $\Re s = -1$, the functional equation moves ξ'/ξ to $\Re(1-s) = 2$, where it has logarithmic growth; Gaussian decay gives the stated remainder. □

5 Closed form of the arithmetic object

The first StrandLoop pass reveals that the object in (9) has an exact elementary smoothing representation.

Theorem 5.1 (Error-function kernel closure)**.** *For $x > 0$,*

$$\boxed{K_\alpha(x) = \frac{e^{\alpha/4}}{2}\,\mathrm{erfc}\!\left(\frac{\alpha - \log x}{2\sqrt{\alpha}}\right)}. \tag{12}$$

If $\mathcal{K}_\alpha(y) := K_\alpha(e^y)$, then

$$\boxed{\mathcal{K}'_\alpha(y) = \frac{1}{2\sqrt{\pi\alpha}} e^{y/2 - y^2/(4\alpha)} = \frac{e^{\alpha/4}}{2\sqrt{\pi\alpha}} e^{-(y-\alpha)^2/(4\alpha)}}. \tag{13}$$

In particular $K_\alpha(0^+) = 0$ and $K_\alpha(+\infty) = e^{\alpha/4}$.

Proof. Differentiate (8) after writing $x = e^y$. The factor s cancels, so the contour may be shifted to $\Re s = 1/2$ without crossing a singularity. With $s = 1/2 + i\xi$,

$$\mathcal{K}'_\alpha(y) = \frac{e^{y/2}}{2\pi} \int_{\mathbb{R}} e^{-\alpha\xi^2} e^{i\xi y}\, d\xi,$$

which is (13) by the Gaussian Fourier integral. On the original line $c > 1$, $K_\alpha(x) = O_{\alpha,c}(x^c)$ as $x \to 0^+$, so the integration constant is fixed by $K_\alpha(0^+) = 0$. Integrating (13) from $-\infty$ to $\log x$ gives (12). □

Proposition 5.2 (Exact PNT-error reduction)**.** *Let*

$$E(x) := \psi(x) - x.$$

Then for $t > 1$,

$$\int_0^\infty K_\alpha(t/x)\, dx = e^{\alpha/4} t, \tag{14}$$

and

$$\mathcal{P}^\circ_\alpha(t) = t^{-1/2} \int_{0^-}^\infty K_\alpha(t/x)\, dE(x) \tag{15}$$

$$= t^{-1/2} \int_0^\infty \frac{E(x)}{x} \mathcal{K}'_\alpha\left(\log \frac{t}{x}\right)\, dx. \tag{16}$$

Proof. The substitution $z = t/x$ followed by integration by parts in logarithmic coordinates and (13) gives (14). The von Mangoldt sum in (9) is the Stieltjes integral of $K_\alpha(t/x)$ against $d\psi(x)$; subtracting (14) gives (15). The endpoint terms vanish and

$$\frac{d}{dx} K_\alpha(t/x) = -\frac{1}{x} \mathcal{K}'_\alpha\left(\log \frac{t}{x}\right),$$

which gives (16). □

Theorem 5.3 (Gaussian heat-flow representation)**.** *Set*

$$F(v) := e^{-v/2}\big(\psi(e^v) - e^v\big), \qquad h_\alpha(y) := \frac{1}{2\sqrt{\pi\alpha}} e^{-y^2/(4\alpha)}. \tag{17}$$

Then for every real u for which the field is defined,

$$\boxed{\mathcal{P}^\circ_\alpha(e^u) = (h_\alpha * F)(u) = \frac{1}{2\sqrt{\pi\alpha}} \int_{\mathbb{R}} e^{-(u-v)^2/(4\alpha)} e^{-v/2}\big(\psi(e^v) - e^v\big)\, dv}. \tag{18}$$

Accordingly we may define, pointwise,

$$\boxed{\mathcal{P}^\circ_\alpha(e^u) = e^{\alpha\partial_u^2} F(u) := h_\alpha * F(u)}. \tag{19}$$

Proof. Put $t = e^u$ and $x = e^v$ in (16). Equation (13) gives

$$e^{-u/2}\mathcal{K}'_\alpha(u-v) = h_\alpha(u-v)e^{-v/2},$$

and (18) follows. $\square$

Remark 5.4 (Unnormalized heat-flow firewall). With $\widehat{f}(\xi) = \int f(u)e^{-iu\xi}\,du$ one has $\widehat{h}_\alpha(\xi) = e^{-\alpha\xi^2}$. The pointwise identity $\mathcal{P}^\circ_\alpha(e^u) = h_\alpha * F(u)$ is unconditional, but the elementary bound on ψ does not place the unnormalized F in $\mathcal{S}'$. Therefore the global Fourier identity for F is not used unconditionally.

Theorem 5.5 (Tempered half-normalization)**.** *Define*

$$\widetilde{F}(v) := e^{-v/2}F(v) = e^{-v}\big(\psi(e^v) - e^v\big), \qquad \widetilde{\mathcal{P}^\circ_\alpha}(u) := e^{-u/2}\mathcal{P}^\circ_\alpha(e^u). \tag{20}$$

Then

$$|\widetilde{F}(v)| \ll 1 + |v|, \qquad \widetilde{F} \in L^1_{\mathrm{loc}}(\mathbb{R}) \cap \mathcal{S}'(\mathbb{R}). \tag{21}$$

If

$$k_\alpha(u) := e^{-u/2}h_\alpha(u) = e^{\alpha/4}h_\alpha(u+\alpha), \tag{22}$$

then

$$\boxed{\widetilde{\mathcal{P}^\circ_\alpha} = k_\alpha * \widetilde{F} = e^{\alpha/4}(h_\alpha * \widetilde{F})(\,\cdot + \alpha\,)}. \tag{23}$$

Moreover $k_\alpha \in \mathcal{S}(\mathbb{R})$ *and*

$$\boxed{\widehat{k_\alpha}(\xi) = m_\alpha(\xi) := e^{\alpha/4 - \alpha\xi^2 + i\alpha\xi}, \qquad \widehat{\widetilde{\mathcal{P}^\circ_\alpha}} = m_\alpha \widehat{\widetilde{F}}.} \tag{24}$$

In particular $\widetilde{\mathcal{P}^\circ_\alpha} \in C^\infty(\mathbb{R}) \cap \mathcal{O}_M(\mathbb{R}) \subset \mathcal{S}'(\mathbb{R})$*.*

Proof. For $v < \log 2$, $\psi(e^v) = 0$, so $\widetilde{F}(v) = -1$. For $v \geq \log 2$,

$$0 \leq \psi(e^v) \leq \sum_{n \leq e^v} \log n \leq e^v v,$$

which gives (21). Multiplying (18) by $e^{-u/2}$ and using $F(v) = e^{v/2}\widetilde{F}(v)$ gives (23). The Gaussian identity in (22) follows by completing the square. Fourier transformation in $\mathcal{S}'$ gives (24). Convolution of a Schwartz function with a tempered distribution is smooth with derivatives of polynomial growth. $\square$

Proposition 5.6 (Exact Gaussian range and safe deconvolution)**.** *Let*

$$\mathcal{R}_\alpha := m_\alpha \mathcal{S}'(\mathbb{R}) = \big\{G \in \mathcal{S}'(\mathbb{R}) : m_\alpha^{-1}G \in \mathcal{S}'(\mathbb{R}) \text{ in } \mathcal{D}'(\mathbb{R})\big\}. \tag{25}$$

Multiplication by m_α *is injective on* $\mathcal{S}'$*, and*

$$\widehat{\widetilde{\mathcal{P}^\circ_\alpha}} \in \mathcal{R}_\alpha, \qquad \widehat{\widetilde{F}} = m_\alpha^{-1}\widehat{\widetilde{\mathcal{P}^\circ_\alpha}}.$$

The inverse is understood only on $\mathcal{R}_\alpha$*. Since*

$$m_\alpha^{-1}(\xi) = e^{-\alpha/4 + \alpha\xi^2 - i\alpha\xi}$$

is not a tempered multiplier, no global backward-heat operator on all of $\mathcal{S}'$ *is asserted.*

Proof. The multiplier m_α is smooth and nowhere zero. If $m_\alpha T = 0$, localization to any compact set permits multiplication by the smooth reciprocal there, so $T = 0$. The range characterization is then tautological. The reciprocal has Gaussian growth and is not a multiplier of $\mathcal{S}'$ in general. $\square$

Corollary 5.7 (Entire regularization)**.** *The function $\widetilde{\mathcal{P}^\circ_\alpha}$ extends to an entire function of $z = x + iy$ satisfying*

$$|\widetilde{\mathcal{P}^\circ_\alpha}(x+iy)| \ll_\alpha (1+|x|)e^{y^2/(4\alpha)}.$$

For each fixed $j \geq 0$ its jth derivative satisfies a bound of the form

$$|\partial_z^j \widetilde{\mathcal{P}^\circ_\alpha}(x+iy)| \ll_{\alpha,j} (1+|x|+|y|^j)e^{y^2/(4\alpha)}.$$

Remark 5.8 (What CPM now asks). The arithmetic gate remains a local L^2 question, but v1.4 supplies two exact descriptions: the original pointwise heat flow of F and the unconditional tempered heat flow of $\widetilde{F}$. The latter is the correct object for global Fourier and range arguments.

6 Positive atomic source and exact bridge to the v1.4 normalization

The v1.4 source $\widetilde{F}$ is a centered cumulative object. The present section extracts the underlying positive atomic source without changing the contour identity or the record-edge capture branch.

Definition 6.1 (Positive arithmetic source)**.** *Define the Radon measure*

$$\boxed{M_\psi := \sum_{n\geq 2} \frac{\Lambda(n)}{n}\,\delta_{\log n}.} \tag{26}$$

Equivalently, under the logarithmic change of variables,

$$M_\psi = e^{-v}\,d\psi(e^v). \tag{27}$$

Proposition 6.2 (Unconditional temperedness of the positive source)**.** *The measure M_ψ is positive and tempered:*

$$M_\psi \in \mathcal{M}_{+,\mathrm{temp}}(\mathbb{R}) \subset \mathcal{S}'(\mathbb{R}).$$

More concretely, for $R \geq 1$,

$$M_\psi([0,R]) = \sum_{2\leq n\leq e^R} \frac{\Lambda(n)}{n} \ll 1 + R^2.$$

Proof. Positivity is immediate from $\Lambda(n) \geq 0$. Since $\Lambda(n) \leq \log n$,

$$\sum_{2\leq n\leq e^R} \frac{\Lambda(n)}{n} \leq \sum_{2\leq n\leq e^R} \frac{\log n}{n} \ll 1 + R^2$$

by comparison with $\int_1^{e^R} (\log x)x^{-1}\,dx$. There is no mass below $\log 2$. Hence M_ψ has polynomial mass growth and defines a tempered positive measure; see, e.g., Schwartz [7] or Hörmander [8] for the distribution-theoretic framework. $\square$

Proposition 6.3 (Exact bridge from the centered source to the positive source)**.** *Let*

$$A(v) := e^{-v}\psi(e^v) = \widetilde{F}(v) + 1.$$

Then in $\mathcal{D}'(\mathbb{R})$*,*

$$\boxed{M_\psi = (\partial_v + 1)A = (\partial_v + 1)(\widetilde{F} + 1) = \partial_v \widetilde{F} + \widetilde{F} + 1.} \tag{28}$$

Proof. In Stieltjes form,

$$dA = d\big(e^{-v}\psi(e^v)\big) = -A(v)\,dv + e^{-v}\,d\psi(e^v).$$

The second term is exactly the atomic measure in (27). Therefore $dA + A\,dv = M_\psi$, which is (28) in distribution notation. □

Remark 6.4 (Why this bridge matters)*.* The positive source M_ψ is not introduced as a replacement for the v1.4 centered source. Equation (28) proves that the two descriptions are linked by an exact local distributional operator. Hence the intrinsic-source branch remains attached to the same arithmetic data rather than importing an unrelated positive measure.

7 Gaussian source injectivity and the intrinsic positive-definite invariant

For this section write $G_\alpha := h_\alpha$, so that

$$G_\alpha(v) = \frac{1}{\sqrt{4\pi\alpha}} e^{-v^2/(4\alpha)}, \qquad \widehat{G}_\alpha(\xi) = e^{-\alpha\xi^2}$$

under the Fourier convention already used in Section 5. Define

$$\boxed{Q_\alpha := G_\alpha * M_\psi.} \tag{29}$$

Theorem 7.1 (Injectivity of Gaussian smoothing on tempered sources)**.** *For every $\alpha > 0$, the map*

$$\mathcal{G}_\alpha : \mathcal{S}'(\mathbb{R}) \to \mathcal{S}'(\mathbb{R}), \qquad T \mapsto G_\alpha * T,$$

is injective. In particular, Q_α uniquely determines the actual arithmetic source M_ψ within $\mathcal{S}'(\mathbb{R})$.

Proof. If $G_\alpha * T = 0$, Fourier transformation gives

$$e^{-\alpha\xi^2}\widehat{T} = 0 \qquad \text{in } \mathcal{D}'(\mathbb{R}).$$

For any compactly supported test function χ, the product $\chi(\xi)e^{\alpha\xi^2}$ is again smooth and compactly supported. Hence

$$\langle \widehat{T}, \chi \rangle = \left\langle e^{-\alpha\xi^2}\widehat{T}, \chi e^{\alpha\xi^2} \right\rangle = 0.$$

Thus $\widehat{T} = 0$, and Fourier injectivity on $\mathcal{S}'$ gives $T = 0$. This is the same localized-reciprocal mechanism used in Proposition 5.6; standard background is in Hörmander [8]. □

Remark 7.2 (Uniqueness is not stability)*.* The theorem is an identifiability statement, not a global well-posedness statement. Formally,

$$\widehat{M_\psi} = e^{\alpha\xi^2}\widehat{Q}_\alpha,$$

but $e^{\alpha\xi^2}$ is not a multiplier of $\mathcal{S}'$ and amplifies high-frequency perturbations super-exponentially.[2]

[2] This distinction is essential throughout v1.5: the proof program should extract exact intrinsic consequences of range membership rather than rely on numerically stable backward heat evolution.

Definition 7.3 (Positive-definite tempered distributions). *A tempered distribution $U \in \mathcal{S}'(\mathbb{R})$ is* positive definite *if*

$$\langle U, \varphi * \widetilde{\varphi} \rangle \geq 0 \qquad (\varphi \in \mathcal{S}(\mathbb{R})), \quad \widetilde{\varphi}(x) := \overline{\varphi(-x)}.$$

Write $\mathcal{S}'_{\mathrm{pd}}(\mathbb{R})$ for this cone.

By the Bochner–Schwartz theorem, the Fourier transform of a positive tempered measure is a positive-definite tempered distribution, and conversely [6, 7, 9]. Since Proposition 6.2 gives $M_\psi \geq 0$, we obtain $\widehat{M_\psi} \in \mathcal{S}'_{\mathrm{pd}}(\mathbb{R})$.

Definition 7.4 (Intrinsic source invariant). *For a Gaussian source trajectory Q_α, set*

$$\mathfrak{U}_\alpha(Q) := \left\{ U \in \mathcal{S}'_{\mathrm{pd}}(\mathbb{R}) : \widehat{Q}_\alpha(\xi) = e^{-\alpha\xi^2} U(\xi) \right\}. \tag{30}$$

Theorem 7.5 (α-independent positive source invariant). *For the arithmetic trajectory* (29)*,*

$$\boxed{\mathfrak{U}_\alpha(Q) = \{\widehat{M_\psi}\} \qquad (\alpha > 0).} \tag{31}$$

In particular, $\mathfrak{U}_\alpha(Q)$ is a singleton and is independent of the smoothing scale.

Proof. The defining convolution gives

$$\widehat{Q}_\alpha = e^{-\alpha\xi^2} \widehat{M_\psi}.$$

Thus $\widehat{M_\psi} \in \mathfrak{U}_\alpha(Q)$. If another U satisfies the same identity, then

$$e^{-\alpha\xi^2}(U - \widehat{M_\psi}) = 0.$$

The localized reciprocal argument from Theorem 7.1 gives $U = \widehat{M_\psi}$. Independence of α is immediate because the recovered source is the same M_ψ for every member of the heat trajectory. □

Corollary 7.6 (Semigroup consistency invariant). *For all $\alpha, \beta > 0$,*

$$Q_{\alpha+\beta} = G_\beta * Q_\alpha, \qquad \widehat{Q}_{\alpha+\beta} = e^{-\beta\xi^2} \widehat{Q}_\alpha,$$

and therefore

$$\mathfrak{U}_{\alpha+\beta}(Q) = \mathfrak{U}_\alpha(Q) = \{\widehat{M_\psi}\}.$$

8 A6 gauge quotient: what Q cannot identify

The preceding section concerns injectivity of the arithmetic source map. It must not be conflated with identifiability of an auxiliary spectral decomposition.

Definition 8.1 (Diagonal gauge quotient). *Let $\mathscr{X}$ be an admissible class of pairs (R, S) and let $\mathscr{T}$ be a nontrivial class of compatible fields such that*

$$(R, S) \in \mathscr{X}, \ T \in \mathscr{T} \quad \Longrightarrow \quad (R + T, S + T) \in \mathscr{X}.$$

Define

$$\pi(R, S) := R - S, \qquad (R, S) \sim (R + T, S + T).$$

Theorem 8.2 (Gauge non-identifiability for Q-factored observers). *Suppose an observer has the form*

$$\mathcal{O} = \Phi \circ \pi.$$

If an admissible gauge orbit contains (R, S) and $(R+T, S+T)$ with $T \neq 0$, then no reconstruction map depending only on $\mathcal{O}$ can recover S on the whole orbit.

Proof. Gauge invariance gives

$$\mathcal{O}(R+T, S+T) = \Phi((R+T) - (S+T)) = \Phi(R-S) = \mathcal{O}(R, S).$$

If $\mathcal{R}(\mathcal{O}(R, S)) = S$ held throughout the orbit, the same input would also have to satisfy

$$\mathcal{R}(\mathcal{O}(R+T, S+T)) = S+T,$$

forcing $S = S + T$, contrary to $T \neq 0$. □

Corollary 8.3 (Information split). *The two maps*

$$(R, S) \mapsto Q := R - S, \qquad M_\psi \mapsto Q_\alpha := G_\alpha * M_\psi$$

have different identifiability behavior: the first is noninjective on any nontrivial diagonal gauge orbit, while the second is injective on $\mathcal{S}'(\mathbb{R})$. Consequently, failure to identify the spectral bookkeeping component S from Q does not imply loss of the intrinsic arithmetic source.

Remark 8.4 (Observer-kernel firewall). A zero output or coincident outputs under an observer do not establish absence of the underlying signal unless the observer is known to be injective on the relevant class. This distinction is now treated as a permanent audit rule.[3]

9 Arithmetic-separator interface

Theorems 7.1, 7.5, and 8.2 identify the surviving object but do not prove RH. Generic positive sources already produce positive-definite α-independent invariants. A successful unconditional route must therefore use arithmetic structure beyond generic positivity.

Define the generic positive Gaussian cone

$$\mathcal{C}_{+,\alpha} := \{G_\alpha * \mu : \mu \in \mathcal{M}_{+,\mathrm{temp}}(\mathbb{R})\}. \tag{32}$$

The actual arithmetic trajectory is the distinguished element generated by M_ψ. Positivity alone characterizes too large a class to be accepted as an RH discriminator.

Unproved Closure Placeholder 9.1 (Arithmetic separator Σ). *Construct a mathematically explicit functional, family of functionals, matrix hierarchy, cone-face invariant, or other intrinsic object*

$$\Sigma : \mathcal{S}'_{\mathrm{pd}}(\mathbb{R}) \longrightarrow \mathscr{Y}$$

together with a precisely defined arithmetic admissible set $\mathscr{K}_{\mathrm{arith}} \subseteq \mathscr{Y}$ satisfying all five requirements:

[3] The rule arose operationally after several failed routes were found to be failures of an observer or quotient rather than proofs that the underlying local object vanished. The theorem above is the exact algebraic form needed here.

(S1) ***Unconditional arithmetic validity:***

$$\Sigma(\widehat{M_\psi}) \in \mathscr{K}_{\mathrm{arith}}$$

is proved from the prime source without assuming RH.

(S2) ***Intrinsic/gauge-safe:*** *Σ depends only on the surviving source invariant $\widehat{M_\psi}$ (equivalently on the exact Gaussian trajectory) and does not require choosing R and S inside a gauge fiber.*

(S3) ***Strictly stronger than generic positivity:*** *the condition is not automatic for every $\mu \in \mathcal{M}_{+,\mathrm{temp}}$; in particular, the proposed separator must distinguish a proper subclass of the generic positive cone.*

(S4) ***Off-line incompatibility:*** *from a hypothetical zero ρ with $\Re\rho \neq \frac{1}{2}$, an exact explicit-formula bridge forces*

$$\Sigma(\widehat{M_\psi}) \notin \mathscr{K}_{\mathrm{arith}}.$$

(S5) ***No hidden RH-equivalence:*** *every hypothesis used in (S1)–(S4) is proved unconditionally or is separately named as an open lemma; no form of RH, zero simplicity, pair correlation, or unproved density estimate is smuggled into the separator.*

Research Gate 9.2 (Counterexample / kill test for candidate separators)**.** *For every proposed finite or infinite family of intrinsic positive-source constraints, attempt to construct a non-arithmetic $\mu \in \mathcal{M}_{+,\mathrm{temp}}$ whose Gaussian trajectory satisfies the same constraints. If such a μ exists, that candidate family does not separate the arithmetic source from the generic positive cone and is killed. If no such μ can exist, identify and prove the exact obstruction.*

Unproved Closure Placeholder 9.3 (Exact zero–source bridge)**.** *Starting from the canonical explicit formula already proved in Section 5, derive an exact identity or inequality that takes a hypothetical off-line zero orbit to a violation of the arithmetic separator in Placeholder 9.1. The bridge must retain multiplicity, conjugation, the functional-equation mirror, contour/pole terms, and every smoothing remainder at the scale used by Σ.*

Unproved Closure Placeholder 9.4 (Unconditional RH closure)**.** *Prove Placeholders 9.1 and 9.3. Then the same object $\widehat{M_\psi}$ would be forced both to satisfy and to violate $\mathscr{K}_{\mathrm{arith}}$ under $\neg$RH, yielding*

$$\neg\mathrm{RH} \Longrightarrow \bot, \qquad \textit{hence} \qquad \mathrm{RH}.$$

This box is deliberately a placeholder, not a theorem.

Remark 9.5 (Relation to the five v1.4 gates)*.* The separator program is a parallel intrinsic-source branch and is *not* silently installed as a sixth record-edge gate. GP/LB/QB/CPM/SI retain their v1.4 meanings. The two branches may later meet through the explicit formula, but a failure in one branch is not repaired by changing the definitions in the other.

10 Defect–compensator closure geometry

The new audit layer does not replace either proof branch. It asks a narrower structural question at a wall: after a hypothesized bad configuration creates a defect, can the admissible remaining degrees of freedom reproduce that defect exactly, approximately, or not even in closure?

Definition 10.1 (Closed compensator range and bore margin)**.** *Let Y be a normed space, let C be a linear compensator domain, let $B : C \to Y$ be linear, and let $b \in Y$ be a defect. Put*

$$\mathcal{M}_B := \overline{B(C)}^{Y}, \qquad \delta_Y(b) := \operatorname{dist}(b, \mathcal{M}_B).$$

The three distinct regimes are

$$\begin{aligned} -b \in B(C) \quad &: \quad \textit{exact compensation},\\ -b \in \mathcal{M}_B \setminus B(C) \quad &: \quad \textit{approximate compensation / closure trap},\\ b \notin \mathcal{M}_B \quad &: \quad \textit{positive separation / bore}. \end{aligned}$$

Theorem 10.2 (Quotient/dual bore criterion)**.** *Let Y be a normed space and $\mathcal{M} \subset Y$ a closed linear subspace. For every $b \in Y$,*

$$\operatorname{dist}(b, \mathcal{M}) = \sup_{\substack{\Lambda \in Y^* \\ \Lambda|_{\mathcal{M}} = 0 \\ \|\Lambda\| \le 1}} |\Lambda(b)|. \tag{33}$$

Consequently, for $\mathcal{M} = \overline{B(C)}$,

$$\boxed{\delta_Y(b) > 0 \iff \exists \Lambda \in Y^* : \; B^*\Lambda = 0, \quad \Lambda(b) \neq 0.} \tag{34}$$

If Y is Hilbert, then

$$\delta_Y(b) = \|P_{\mathcal{M}^\perp} b\|_Y.$$

Proof. Pass to the quotient $Y/\mathcal{M}$. Its norm is exactly $\|[b]\| = \operatorname{dist}(b, \mathcal{M})$. Hahn–Banach gives a norm-one functional on the quotient attaining this norm on the one-dimensional span of $[b]$; composing with the quotient map gives the upper bound in (33) with equality. Conversely every norm-one functional vanishing on $\mathcal{M}$ satisfies $|\Lambda(b)| \le \operatorname{dist}(b, \mathcal{M})$. The equivalence (34) follows because $B^*\Lambda = 0$ is equivalent to vanishing on $B(C)$ and hence on its closure. The Hilbert statement is the orthogonal-projection theorem. □

Remark 10.3 (Closure firewall). The implication

$$b \notin B(C) \Longrightarrow \delta_Y(b) > 0$$

is false in infinite dimension. A proper algebraic range may be dense. This is the central distinction the numerical program must preserve.

Proposition 10.4 (One-sided logical firewall)**.** *Suppose A is already proved unconditionally for the fixed arithmetic source and* OFF *denotes the existence of an off-line zero. Then, over the established truth of A,*

$$(\mathrm{OFF} \Rightarrow \neg A) \iff \neg\mathrm{OFF}.$$

Thus a purported bore that factors completely through a static predicate of the source alone has not reduced the logical distance to RH; a genuine v1.6 bore must keep both defect and compensator data load-bearing in the final relational observable.

Proof. With A true, the implication $\mathrm{OFF} \Rightarrow \neg A$ is equivalent to $\neg\mathrm{OFF}$ by elementary propositional logic. □

11 Prime-ray Euler closure as a certified negative control

The first full infinite-dimensional closure test is deliberately negative. It calibrates both the mathematics and the numerics before the new lower-block observer is trusted.

Theorem 11.1 (Prime-ray Euler density in $C(I)$)**.** *Let $I = [\sigma_0, \sigma_1]$ with $0 < \sigma_0 < \sigma_1 < \infty$, let*

$$g_p(s) := \frac{1}{p^{s+1} - 1}, \qquad C_0 := \mathbb{R}^{(\mathbb{P})},$$

and define

$$B : C_0 \to C(I), \qquad (Bc)(s) = \sum_p c_p g_p(s).$$

Then

$$\boxed{\overline{B(C_0)}^{C(I)} = \overline{\operatorname{span}_{\mathbb{R}}\{g_p : p \in \mathbb{P}\}}^{C(I)} = C(I).} \tag{35}$$

Equivalently,

$$\boxed{\ker B^* = \{0\}.}$$

The algebraic range is nevertheless proper. In particular,

$$1 \notin B(C_0), \qquad 1 \in \overline{B(C_0)}^{C(I)}, \qquad \delta_{C(I)}(1) = 0.$$

Proof. By the Riesz representation theorem, an annihilator of the span has the form

$$\Lambda(f) = \int_I f(s)\, d\nu(s)$$

for a finite signed measure ν. Put $a = 1 + \sigma_0$ and define

$$L_\nu(z) := \int_I e^{-(s+1)z}\, d\nu(s), \qquad F_\nu(z) := \int_I \frac{d\nu(s)}{e^{(s+1)z} - 1} \quad (\Re z > 0).$$

Absolute convergence gives

$$F_\nu(z) = \sum_{m \geq 1} L_\nu(mz), \qquad |F_\nu(z)| \leq \frac{\|\nu\|_{\mathrm{TV}}}{e^{a\Re z} - 1}.$$

Choose c with $(\log 2)/a < c < \log 2$. Then $F_\nu \in H^\infty(\{\Re z > c\})$. If $\Lambda(g_p) = 0$ for every prime, then $F_\nu(\log p) = 0$ for every prime. The half-plane Blaschke condition for zeros of a nonzero bounded analytic function would require convergence of

$$\sum_p \frac{\log p - c}{1 + (\log p - c)^2},$$

but this diverges, equivalently to the classical divergence of $\sum_p 1/\log p$. Hence $F_\nu \equiv 0$ on the half-plane; see, e.g., Duren [10] for the disk theorem and its conformally equivalent half-plane form.

To recover L_ν, define

$$(\mathscr{K} L)(z) := \sum_{m \geq 2} L(mz).$$

On the weighted analytic space with norm

$$\|L\|_{a,c} := \sup_{\Re z \geq c} e^{a\Re z} |L(z)|,$$

one has

$$\|\mathscr{K}\| \le \frac{e^{-ac}}{1-e^{-ac}} < 1.$$

Thus $I+\mathscr{K}$ is invertible; equivalently, absolute Möbius inversion gives

$$L_\nu(z) = \sum_{n\ge 1} \mu(n) F_\nu(nz) = 0.$$

Since L_ν is entire,

$$0 = L_\nu^{(k)}(0) = (-1)^k \int_I (s+1)^k \, d\nu(s) \qquad (k \ge 0).$$

Polynomial density on I forces $\nu = 0$. Hence the annihilator is trivial and (35) follows by Hahn–Banach.

Finally, the functions g_p are linearly independent, so the finite-support range is countable-dimensional and proper. More concretely, if a finite combination equaled the constant 1 on I, analyticity would extend the identity to $s > 0$, whereas every such finite combination tends to 0 as $s \to +\infty$. Density then gives the stated closure trap for the constant target. □

Remark 11.2 (Numerical calibration rule). Every finite truncation B_N can leave a positive residual against a target even though the true closure distance is zero. Therefore the Euler experiment is a mandatory negative control: a finite-dimensional pipeline is not trusted unless enrichment of the prime-ray basis drives its normalized residual toward the certified dense-range behavior rather than manufacturing a stable false bore.

12 Regime I: signed lower block in $L^2([0,\infty))$

The exact source bridge already chooses a canonical primitive

$$A(v) := \widetilde{F}(v) + 1 = e^{-v}\psi(e^v), \qquad M_\psi = (\partial_v + 1)A.$$

If one instead solves $(\partial_v + 1)m = M_\psi$ abstractly, a homogeneous term Ce^{-v} appears. No such ambiguity is introduced when A is defined directly from the already fixed $\widetilde{F}$. For the closure program, however, A is not the signed square-integrable object; the centered remainder is

$$\boxed{r(v) := A(v) - 1 = \widetilde{F}(v).}$$

Proposition 12.1 (Unweighted Regime-I admissibility)**.** *On* $[0,\infty)$,

$$\boxed{r \in L^2([0,\infty)), \qquad A \notin L^2([0,\infty)).}$$

Proof. The classical unconditional prime-number-theorem error estimate gives, for some $c > 0$,

$$\psi(x) - x = O\Big(xe^{-c\sqrt{\log x}}\Big).$$

With $x = e^v$ this becomes

$$r(v) = O(e^{-c\sqrt{v}}) \qquad (v \to +\infty),$$

whose square is integrable. Local square-integrability on bounded intervals is immediate from the definition of $\psi(e^v)$. Thus $r \in L^2([0,\infty))$. The PNT also gives $A(v) = e^{-v}\psi(e^v) \to 1$, so $A \notin L^2([0,\infty))$. Standard forms of the quoted PNT estimate are recorded in Titchmarsh–Heath-Brown [2]. □

Definition 12.2 (Certified exponential-weight family)**.** *For $a \geq 0$ put*

$$Y_a := L^2\big([0,\infty), e^{av}\,dv\big).$$

Version 1.6 calls a parameter a certified admissible *only when membership of the actual arithmetic lower block and every eventual compensator is proved unconditionally from the frozen paper and standard unconditional estimates.*

Remark 12.3 (What $a = 0$ does and does not mean). Proposition 12.1 certifies $a = 0$. The available subexponential PNT upper bound does not by itself prove $r \in Y_a$ for any fixed $a > 0$, but it also does *not* prove $r \notin Y_a$. Accordingly the present paper treats

$$\boxed{a = 0 \text{ as the current certified endpoint of the exponential-weight search.}}$$

In particular, the defect-neutralizing exponential weight suggested by a hypothetical decay rate is not installed merely because it improves visual separation. Any future $a > 0$ must earn admissibility by an independent theorem.

Research Gate 12.4 (Regime-I exact defect/compensator decomposition)**.** *Fix a hypothetical off-line zero*

$$\rho_0 = \beta_0 + i\gamma_0, \qquad \frac{1}{2} < \beta_0 < 1,$$

with its complete multiplicity-safe conjugation and functional-equation orbit. Starting from the paper's exact explicit-formula machinery, derive in

$$Y_0 = L^2([0,\infty))$$

a definition-complete identity

$$\boxed{r = b_{\rho_0} + B_0 c.} \tag{36}$$

The term b_{ρ_0} must be the exact contribution of the selected orbit, not a schematic symbol, and $B_0 c$ must contain every admissible non-target zero term, pole/trivial-zero/background term, and normalization term that lives in the same observer space. The compensator domain C_0 must represent only genuine degrees of freedom already present in the exact formula; arbitrary L^2 functions are forbidden.

Research Gate 12.5 (Regime-I closure/density gate)**.** *After Gate 12.4 is definition-complete, put*

$$\mathcal{M}_0 := \overline{B_0(C_0)}^{Y_0}.$$

Decide the binary closure question

$$\boxed{b_{\rho_0} \overset{?}{\in} \mathcal{M}_0.} \tag{37}$$

If $b_{\rho_0} \notin \mathcal{M}_0$, construct or characterize

$$\Lambda \in Y_0^* \cong Y_0, \qquad B_0^* \Lambda = 0, \qquad \Lambda(b_{\rho_0}) \neq 0.$$

If $b_{\rho_0} \in \mathcal{M}_0$, freeze the resulting closure trap and change observer geometry only for an independently justified reason.

Remark 12.6 (Hardy-space route is conditional on the exact compensator form). The Laplace transform is a natural unitary model for $L^2([0,\infty))$ and may convert genuine exponential compensator modes into Hardy-space reproducing kernels. No completeness or Blaschke conclusion is promoted here until Gate 12.4 proves that the actual compensator range has precisely the required form. Phase locking alone is not an annihilator theorem.

13 Internal wall survey and data-first optimization program

The defect–compensator framework is first tested against this paper's own walls. Version 1.6 does not claim a universal obstruction engine and does not import unrelated mathematical examples. The survey population is

$$\boxed{\text{GP, LB, QB, CPM, SI, EULER–CI, BLC}_0,}$$

where BLC_0 denotes the Regime-I lower-block closure problem in Y_0. The historical gate name LB is reserved for the record-edge lower-block separation problem and is never reused for BLC_0.

Interface	Meaning in this paper	v1.6 status
GP	Gaussian persistence of a fresh record edge	Open live gate
LB	Record-edge lower finite-block separation	Open live gate
QB	Quantitative Bohr relative-density control	Open live gate
CPM	Heat-flow centered-prime mean-square bound	Open live gate
SI	Common subsequence/thickness selection	Open live gate
EULER-CI	Prime-ray closure in $C(I)$	Certified dense-range negative control; three-run numerical calibration complete
BLC_0	Regime-I lower-block closure in $L^2([0,\infty))$	Decomposition and closure open

Definition 13.1 (Finite survey margin)**.** *Whenever a wall has a mathematically faithful finite compensator model B_N, define*

$$J_N(b) := \frac{\operatorname{dist}_Y(b, \operatorname{Ran} B_N)}{\|b\|_Y}, \qquad b \neq 0. \tag{38}$$

The survey also records the smallest singular value, effective rank, condition number, a normalized candidate dual residual, the dual signal on b, and witness stability under $N \mapsto N'$. These quantities are diagnostics, not theorem statements.

Remark 13.2 (Finite-dimensional firewall)*.* No statement of the form $J_N > 0$ is evidence by itself for

$$\operatorname{dist}\Big(b, \overline{B(C)}\Big) > 0.$$

The Euler negative control proves why: every finite prime-ray span is proper while the infinite closure is all of $C(I)$. The first required numerical behavior is therefore correct reproduction of the known erosion $J_N \to 0$ in the EULER-CI control.

13.1 Euler-CI numerical calibration milestone

The certified theorem 11.1 supplies an unusually strong numerical control because the correct infinite-dimensional answer is known in advance:

$$\operatorname{dist}_{C(I)}\Big(1, \overline{\operatorname{span}\{g_p : p \in \mathbb{P}\}}\Big) = 0, \qquad \ker B^* = \{0\}.$$

Accordingly, any apparently persistent positive finite-dimensional residual is known a priori to be a numerical or regularization artifact rather than a continuous bore. Three computational runs were used to calibrate the survey machinery against this theorem. They are retained as *instrumentation evidence only*; none changes the analytic statement of Theorem 11.1.

Run 001: double-precision minimax control. On $I = [0.5, 2]$, with 1000 optimization nodes and 10000 validation nodes, the discrete minimax problem was solved for the first

$$N \in \{5, 10, 20, 50, 100, 200\}$$

prime rays. The nominal column count increased by a factor of forty, but the double-precision numerical rank saturated at

$$5,\ 8,\ 8,\ 9,\ 9,\ 9.$$

Meanwhile the condition number grew from approximately 2.27×10^5 at $N = 5$ to the 10^{16}–10^{19} regime once the near-null directions fell below floating-point resolution. The validation sup residual became erratic rather than monotone, ranging from about 4.49×10^{-6} to 5.72×10^{-5} after rank saturation. Thus nominal N is not a faithful progress variable once the resolved rank has stopped increasing.

Run 002: multiprecision retreat. A second run used 50 decimal digits and L^2 normal equations on smaller grids. Because this is a different optimization problem, its sup residuals are not compared as if they were minimax optima. The decisive calibration is the *precision response*: at $N = 20$ the validation sup residual fell to

$$3.04 \times 10^{-13}, \qquad J_{L^2} = 6.61 \times 10^{-14},$$

while the coefficient norms increased to

$$\|c\|_2 \approx 2.08 \times 10^{16}, \qquad \|c\|_\infty \approx 1.22 \times 10^{16}.$$

The $N = 50$ multiprecision solve returned the zero coefficient vector with residual 1 and is therefore classified as solver breakdown, not mathematical evidence. Run 002 establishes the characteristic dense-range signature

$$\boxed{\text{higher precision} \implies \text{apparent gap retreats while compensator coefficients blow up.}}$$

Run 003: regularization bias. A third double-precision experiment solved

$$(G^T G + \lambda I)c = G^T b, \qquad 10^{-16} \le \lambda \le 10^2,$$

over a 30-point logarithmic regularization grid. The value labelled "best" minimizes validation residual on this grid; it is not claimed to be the geometric L-curve corner. For $N = 20$ through 200, weak regularization stabilizes coefficient norms near 10^4–10^5 while leaving an apparently persistent validation sup residual of order 10^{-5}, for example

$$J_{\infty,\text{val}}(100) \approx 6.34 \times 10^{-6}, \qquad J_{\infty,\text{val}}(200) \approx 8.55 \times 10^{-6}.$$

Since the analytic closure gap is exactly zero, this plateau is a controlled example of *regularization-induced false separation.*

Definition 13.3 (Calibrated pseudo-gap signature)**.** *For this paper's numerical closure survey, the Euler-CI control defines three empirically observed warning mechanisms:*

(P1) ***raw precision floor:*** *effective-rank saturation, near-null singular directions, coefficient growth, and an erratic residual floor;*

(P2) ***precision-retreat/blow-up:*** *increasing arithmetic precision drives the apparent residual sharply downward while the compensator coefficient norm grows by many orders of magnitude;*

(P3) ***regularization bias:*** *stabilizing the coefficient norm can create a persistent positive residual plateau even when the true closure gap is zero.*

No future wall is permitted to infer positive closure distance from a finite residual until these three mechanisms have been ruled out at the relevant operator, topology, discretization, and precision.

Remark 13.4 (Effective rank and precision stability). The primary computational enrichment variable is henceforth the *resolved effective rank*, not nominal basis size N. A future candidate bore must remain quantitatively stable while effective rank increases, arithmetic precision increases, discretization is refined, the admissible compensator family is enlarged, and regularization is weakened or removed. The Euler numerical scales above are not universal thresholds for any other observer space.

Frozen calibration status. The Euler-CI computational campaign is complete for the present architecture. Its role is now purely calibrational. The files `EULER_CI_JN_trajectory.csv`, `EULER_CI_JN_trajectory_mp50.csv`, and `EULER_CI_lcurve.csv`, together with their run metadata, define the current negative-control dataset. Further Euler computation is deferred unless a later BLC_0 anomaly requires a matched comparison.

Research Gate 13.5 (Paper-wall data survey)**.** *Build a common machine-readable ledger for the seven interfaces above. For each wall record only quantities that are definition-complete in the paper: defect, compensator domain, compensator map, observer space, localization/weight parameters, closure status, finite truncation, J_N trajectory, conditioning, and dual-witness stability. Use* ***`UNDERDEFINED`*** *rather than inventing a missing map. The survey sequence is*

$$\boxed{\begin{array}{c}\textit{exact wall definition} \to \textit{data generation} \to \textit{negative-control validation}\\ \to \textit{KANDy search} \to \textit{candidate phase boundary}\\ \to \textit{analytic theorem or kill.}\end{array}}$$

KANDy is asked to search for persistent geometry shared by the paper's own walls, not to infer RH from finite data. A candidate bore is promoted only after an infinite-dimensional closure theorem or a continuous annihilator theorem is proved.

14 Functional-equation quartet geometry

Proposition 14.1 (Exact hyperbolic quartet decomposition)**.** *Let $\rho = \frac{1}{2} + \delta + i\gamma$ with $\delta > 0$ and $\gamma > 0$. Its quartet is*

$$\rho, \quad 1-\overline{\rho}, \quad \overline{\rho}, \quad 1-\rho.$$

Put

$$A_\rho := \frac{W_\alpha(\rho)}{\rho}, \qquad B_\rho := \frac{W_\alpha(1-\overline{\rho})}{1-\overline{\rho}}.$$

Its total contribution to $\mathcal{S}^{\mathrm{G}}_\alpha(e^u)$ is

$$-2\,\mathrm{Re}\Big[e^{i\gamma u}\left((A_\rho + B_\rho)\cosh(\delta u) + (A_\rho - B_\rho)\sinh(\delta u)\right)\Big]. \tag{39}$$

Proof. The two zeros at ordinate $+\gamma$ contribute $-e^{i\gamma u}(A_\rho e^{\delta u} + B_\rho e^{-\delta u})$; the two at $-\gamma$ contribute the conjugate. Expand into hyperbolic functions. □

Remark 14.2 (Growth and phase). The factor $e^{(\beta-1/2)u}$ is radial growth; $e^{i\gamma u}$ is phase rotation. Record-edge selection acts on the first coordinate, while Bohr recurrence manages interference generated by the second.

15 Finite-height record edges and Gaussian capture

For H above the first zero ordinate define

$$\mathcal{Z}(H) := \{\rho : \xi(\rho) = 0,\ |\operatorname{Im}\rho| \le H\}, \qquad \beta(H) := \max_{\rho\in\mathcal{Z}(H)} \operatorname{Re}\rho.$$

Proposition 15.1 (Record-edge dichotomy)**.** *Assume* ¬RH. *Exactly one of the following occurs:*

(I) $\beta(H) = \beta_* > 1/2$ *for all sufficiently large* H*;*

(II) there are strict record heights $H_n \to \infty$ *such that*

$$\frac{1}{2} < \beta(H_1) < \beta(H_2) < \cdots \uparrow \Theta, \qquad \Theta := \sup_\rho \operatorname{Re}\rho > \frac{1}{2}.$$

Proof. $\beta(H)$ is nondecreasing, bounded by 1, and attained at each finite H. □

Define

$$\mathcal{E}_H := \{\rho \in \mathcal{Z}(H) : \operatorname{Re}\rho = \beta(H)\}$$

and

$$G_H(u) := -\sum_{\rho\in\mathcal{E}_H} \frac{W_\alpha(\rho)}{\rho} e^{i\gamma u}. \tag{40}$$

Then G_H is a nonzero finite trigonometric polynomial and hence Bohr uniformly almost-periodic.

If $\mathcal{Z}(H) \setminus \mathcal{E}_H \ne \varnothing$, set

$$\eta_H := \beta(H) - \max_{\rho\in\mathcal{Z}(H)\setminus\mathcal{E}_H} \operatorname{Re}\rho > 0, \qquad C_H := \sum_{\substack{|\gamma|\le H \\ \operatorname{Re}\rho<\beta(H)}} \left|\frac{W_\alpha(\rho)}{\rho}\right|.$$

Otherwise set $\eta_H = 1$, $C_H = 0$.

Proposition 15.2 (Gaussian finite-height edge decomposition)**.** *For* $u = \log t$*,*

$$\mathcal{S}_\alpha^{\mathrm{G}}(e^u) = e^{(\beta(H)-1/2)u} G_H(u) + \mathcal{R}_{\mathrm{low}}(e^u; H) + \mathcal{R}_{\mathrm{high}}(e^u; H),$$

where

$$|\mathcal{R}_{\mathrm{low}}(e^u; H)| \le C_H e^{(\beta(H)-1/2-\eta_H)u}, \tag{41}$$

$$|\mathcal{R}_{\mathrm{high}}(e^u; H)| \le K_\alpha e^{u/2-\alpha H^2/2}. \tag{42}$$

Proof. Split the absolutely convergent zero sum into the edge, the remaining finite block, and the high tail; use Lemma 4.2 on the latter. □

Choose Bohr constants $q_H, \ell_H, L_H > 0$ such that every interval of length $L_H + 2\ell_H$ contains a $2\ell_H$ interval on which $|G_H| \geq q_H$, and normalize

$$0 < 2\ell_H \leq \log 2. \tag{43}$$

Define

$$U_H^- := \max\left\{0, \frac{1}{\eta_H} \log \frac{4C_H}{q_H}\right\}, \tag{44}$$

$$U_H^+ := \frac{\alpha H^2/2 - \log(4K_\alpha/q_H)}{1 - \beta(H)}. \tag{45}$$

When $C_H = 0$, set $U_H^- = 0$.

Theorem 15.3 (Gaussian record-edge capture criterion)**.** *If*

$$U_H^+ - U_H^- \geq L_H + 2\ell_H, \tag{46}$$

then there is an interval $J_H \subset [U_H^-, U_H^+]$ of length $2\ell_H$ on which

$$|\mathcal{S}_\alpha^{\mathrm{G}}(e^u)| \geq \frac{q_H}{2} e^{(\beta(H)-1/2)u}.$$

If $J_H = [a, a + 2\ell_H]$ and $T_H = e^a$, then

$$\int_{e^{J_H}} |\mathcal{S}_\alpha^{\mathrm{G}}(t)|^2 \, dt \geq c_H T_H^{2\beta(H)}, \tag{47}$$

where

$$c_H := \frac{q_H^2}{4} \frac{e^{4\beta(H)\ell_H} - 1}{2\beta(H)} > 0.$$

Proof. On $u \geq U_H^-$ the lower block is at most one quarter of the edge scale; on $u \leq U_H^+$ the high tail is at most one quarter. Bohr recurrence supplies an interval inside the window where $|G_H| \geq q_H$. The reverse triangle inequality and $t = e^u$ give the claims. □

Definition 15.4 (Window surplus)**.**

$$\mathfrak{W}_{\mathrm{G}}(H) := U_H^+ - U_H^- - (L_H + 2\ell_H).$$

16 Gaussian persistence correction

Version 1.2 emphasized the H^2 high-tail advantage. StrandLoop P01 identifies the countervailing effect: the same Gaussian can attenuate a newly born record edge so strongly that U_H^+ becomes negative.

Definition 16.1 (Edge birth ordinate and persistence surplus)**.** *For fixed H let*

$$r_H := \min_{\rho \in \mathcal{E}_H} |\operatorname{Im} \rho|.$$

Define the Gaussian persistence surplus

$$\Pi_H := \frac{H^2}{2} - r_H^2. \tag{48}$$

Proposition 16.2 (Fresh-record obstruction). *There is an exponent $A > 0$ such that*

$$q_H \le H^A e^{-\alpha r_H^2} \tag{49}$$

for all sufficiently large H, after harmless adjustment of the constant. Consequently

$$U_H^+ \le \frac{\alpha(H^2/2 - r_H^2) + A\log H + O_\alpha(1)}{1-\beta(H)}. \tag{50}$$

In particular,

$$r_H^2 - \frac{H^2}{2} \gg \log H \quad \Longrightarrow \quad U_H^+ < 0 \tag{51}$$

for all sufficiently large H. Hence

$$\liminf_{H\to\infty} \frac{r_H^2}{H^2} > \frac{1}{2} \quad \Longrightarrow \quad U_H^+ < 0$$

eventually, and especially

$$\frac{r_H}{H} \to 1 \quad \Longrightarrow \quad U_H^+ \to -\infty.$$

Proof. The Bohr threshold may be chosen no larger than $\sup|G_H|$, and

$$\sup|G_H| \le \sum_{\rho\in\mathcal{E}_H} \left|\frac{W_\alpha(\rho)}{\rho}\right| \ll H^A e^{-\alpha r_H^2}$$

using the zero count and $|\rho|^{-1} \ll 1$ at positive height. Insert the resulting lower bound for $\log(4K_\alpha/q_H)$ into (45). $\square$

Remark 16.3 (Correction to the original S5 threshold). The statement

$$r_H^2 > H^2/2 + o(H^2) \Longrightarrow U_H^+ < 0$$

is not valid without quantitative control of the error relative to $\log H$. Equation (51) is the corrected sufficient condition.

Remark 16.4 (Record persistence, not cutoff height). A larger H helps only if the current right edge was born sufficiently far below H. Thus a record plateau is not a technical convenience: it is the mechanism by which the high-tail Gaussian can improve while the edge coefficient remains fixed at its earlier ordinate.

Definition 16.5 (Plateau surplus). *Suppose a right-edge value is born at ordinate h and persists unchanged to cutoff $H > h$. Define*

$$\Delta_{\mathrm{G}}(H,h) := H^2 - 2h^2. \tag{52}$$

Remark 16.6 (Correction to the $\sqrt{2}$ heuristic). The exponential competition is

$$e^{-\alpha h^2} \quad \text{versus} \quad e^{-\alpha H^2/2},$$

so the natural leading exponent is

$$\alpha\left(\frac{H^2}{2} - h^2\right) = \frac{\alpha}{2}\Delta_{\mathrm{G}}(H,h).$$

The ratio H/h alone loses the relevant second-order information. One may have $H/h \to \sqrt{2}$ while $H^2 - 2h^2 \to +\infty$.

Research Gate 16.7 (GP: Gaussian record-persistence gate). *Under* $\neg$RH*, produce record-edge plateaus* (h_n, H_n) *with* $H_n \to \infty$ *such that*

(GP1) $\beta(H_n) = \beta(h_n) > 1/2$*;*

(GP2) the effective Gaussian surplus beats all amplitude losses, quantitatively

$$\alpha\left(\frac{H_n^2}{2} - h_n^2\right) + \log q_{H_n} + \alpha h_n^2 - \log K_\alpha \to +\infty;$$

(GP3) equivalently, after any proved polynomial-size control of the residual amplitude factors, it is enough to establish

$$H_n^2 - 2h_n^2 \gg \log H_n;$$

(GP4) the resulting $U^+_{H_n}$ *is positive and leaves room for the lower-block and Bohr costs.*

Remark 16.8. Gate 16.7 deliberately keeps the q_H term. The upper estimate (49) alone cannot be inverted into the lower edge-amplitude estimate required for successful capture.

17 Exact lower-block and recurrence costs

For $C_H > 0$, define

$$B_H := \max\left\{0, \log\frac{4C_H}{q_H}\right\}, \qquad D_H := \frac{\alpha H^2}{2} - \log\frac{4K_\alpha}{q_H}. \tag{53}$$

Then

$$U_H^- = \frac{B_H}{\eta_H}, \qquad U_H^+ = \frac{D_H}{1-\beta(H)}. \tag{54}$$

For $C_H = 0$, take $B_H = 0$.

Proposition 17.1 (Exact lower-block criterion). *If* $B_H > 0$*, then*

$$U_H^- = o(H^2) \iff \frac{\log(4C_H/q_H)}{\eta_H H^2} \to 0. \tag{55}$$

If additionally $\eta_H H^2 \to \infty$*, then* (55) *is equivalent to*

$$\frac{\log(C_H/q_H)}{\eta_H H^2} \to 0.$$

Without $\eta_H H^2 \to \infty$*, the latter implication can fail.*

Proof. Divide (44) by H^2. The extra $\log 4$ contributes $\log 4/(\eta_H H^2)$ and cannot be dropped without control of $\eta_H H^2$. □

Corollary 17.2 (Exact two-sided window inequality). *If* $D_H > 0$*, then before the Bohr waiting cost is included,*

$$U_H^- < U_H^+$$

is equivalent to

$$(1-\beta(H))B_H < \eta_H D_H. \tag{56}$$

Moreover

$$U_H^- = o(U_H^+) \iff \frac{(1-\beta(H))B_H}{\eta_H D_H} \to 0.$$

17.1 Quantitative Bohr control: what remains open

Write

$$G_H(u) = \sum_{j=1}^{m_H} a_{j,H} e^{i\gamma_{j,H} u}.$$

Dirichlet simultaneous approximation supplies short *individual* near-returns of the phase vector, with bounds depending on m_H and the requested phase accuracy. This is useful diagnostic information, but the capture theorem uses a stronger constant L_H: every interval of length L_H must contain a good near-return.

Remark 17.3 (First-return versus relative-density firewall). A bound of the form

$$\exists \tau_H \leq Q^{m_H} : \max_j \left\| \frac{\tau_H \gamma_{j,H}}{2\pi} \right\| \leq Q^{-1}$$

does *not* by itself imply

$$L_H \leq Q^{m_H}.$$

The former controls one simultaneous recurrence; the latter controls the maximum gap between suitable recurrences. Version 1.3 therefore does not promote the StrandLoop S8 bound to a theorem.

Research Gate 17.4 (QB: quantitative Bohr relative density)**.** *For the actual edge frequency vectors $\{\gamma_{j,H}\}_{j \leq m_H}$, prove a bound on the relative-density constant L_H and a compatible superlevel width ℓ_H such that along a record-persistent subsequence*

$$L_H + 2\ell_H = o\big(U_H^+ - U_H^-\big)\,, \tag{57}$$

or at minimum $L_H + 2\ell_H = o(H^2)$ whenever the usable window is of Gaussian order H^2.

Remark 17.5. The quantities m_H, the Diophantine geometry of the ordinate vector, the coefficient profile $a_{j,H}$, and the required phase accuracy now form the explicit recurrence subproblem. Qualitative almost periodicity alone is insufficient for asymptotic admissibility.

18 Admissible forcing sequences

Definition 18.1 (Admissible Gaussian capture sequence)**.** *An admissible sequence consists of heights $H_n \to \infty$ and intervals*

$$J_n = [a_n, a_n + 2\ell_{H_n}] \subset [U_{H_n}^-, U_{H_n}^+]$$

such that, with $T_n = e^{a_n}$ and $\beta_n = \beta(H_n)$,

(A1) $\beta_n > 1/2$ and $\mathfrak{W}_{\mathrm{G}}(H_n) \geq 0$;

(A2) $T_n \to \infty$;

(A3) with

$$c_n := \frac{q_{H_n}^2}{4} \frac{e^{4\beta_n \ell_{H_n}} - 1}{2\beta_n},$$

there is $\varepsilon_n \downarrow 0$ such that

$$c_n^{1/2} \geq T_n^{-\varepsilon_n}.$$

Because $2\ell_{H_n} \leq \log 2$,

$$I_n := e^{J_n} = [T_n, T_n e^{2\ell_{H_n}}] \subset [T_n, 2T_n].$$

Proposition 18.2 (Subsequence energy amplification)**.** *Every admissible sequence satisfies*

$$\|\mathcal{S}_\alpha^{\mathrm{G}}\|_{L^2(I_n)} \geq T_n^{\beta_n - \varepsilon_n}. \tag{58}$$

19 The arithmetic gate in heat-flow coordinates

Since $\mathcal{R}_\alpha^{\mathrm{can}}(t) = O_\alpha(t^{-1/2})$ and $I_n \subset [T_n, 2T_n]$,

$$\|\mathcal{R}_\alpha^{\mathrm{can}}\|_{L^2(I_n)} = O_\alpha(1). \tag{59}$$

Research Gate 19.1 (CPM: centered-prime mean square)**.** *For every admissible sequence, or for a selected admissible subsequence sufficient to cover* ¬RH*, prove*

$$\|\mathcal{P}_\alpha^\circ\|_{L^2(I_n)} \le T_n^{1/2+\delta_n}, \qquad \delta_n \to 0, \tag{60}$$

with fixed α *and explicit constant dependence.*

Proposition 19.2 (Exact logarithmic-coordinate norm relation)**.** *Let*

$$I = [T, Te^{2\ell}], \qquad J = [\log T, \log T + 2\ell].$$

Then

$$\|\mathcal{P}_\alpha^\circ\|_{L^2(I)}^2 = \int_J |\mathcal{P}_\alpha^\circ(e^u)|^2 e^u \, du, \tag{61}$$

and therefore

$$T \int_J |\mathcal{P}_\alpha^\circ(e^u)|^2 \, du \le \|\mathcal{P}_\alpha^\circ\|_{L^2(I)}^2 \le Te^{2\ell} \int_J |\mathcal{P}_\alpha^\circ(e^u)|^2 \, du. \tag{62}$$

If $\ell = o(\log T)$*, then at exponent level*

$$\boxed{\|\mathcal{P}_\alpha^\circ\|_{L^2(I)} \le T^{1/2+\delta+o(1)} \quad \Longleftrightarrow \quad \int_J |\mathcal{P}_\alpha^\circ(e^u)|^2 \, du \le T^{2\delta+o(1)}}. \tag{63}$$

Proof. Use $t = e^u$, so $dt = e^u \, du$, and note $T \le e^u \le Te^{2\ell}$ on J. □

Corollary 19.3 (Heat-flow CPM)**.** *For an admissible sequence, the bounded-width normalization* (43) *implies* $\ell_{H_n} = o(\log T_n)$*. Hence Gate 19.1 is equivalent at exponent level to*

$$\boxed{\int_{J_n} \left| e^{\alpha \partial_u^2} \left[e^{-u/2} \big(\psi(e^u) - e^u \big) \right] \right|^2 du \le T_n^{o(1)}}. \tag{64}$$

Remark 19.4 (The hard object, simplified)*.* Equation (64) is the current arithmetic bottleneck: the local energy of a fixed Gaussian heat smoothing of the critical-scale PNT error on the *same* intervals selected by record-edge capture.

20 Exact local-energy defect

The P02–P03 loop exposed a useful way to package the final exponent comparison without discarding the finite interval-width or edge-amplitude factors.

Proposition 20.1 (Prime local-energy defect inequality)**.** *Let* $J_H = [\log T_H, \log T_H + 2\ell_H]$ *be a capture interval on which the canonical remainder is negligible in the sense of* (73)*. Then*

$$\|\mathcal{P}_\alpha^\circ\|_{L^2([T_H, T_H e^{2\ell_H}])}^2 \ge \frac{q_H^2}{16} T_H^{2\beta(H)} \frac{e^{4\beta(H)\ell_H} - 1}{2\beta(H)}. \tag{65}$$

For $\delta_H \geq 0$ define the defect

$$\mathcal{D}_H(\delta_H) := \log \|\mathcal{P}_\alpha^\circ\|^2_{L^2([T_H, T_H e^{2\ell_H}])} - (1 + 2\delta_H) \log T_H \tag{66}$$

and the finite-window penalty

$$\mathcal{L}_H^{\mathrm{loc}} := \log \left[\frac{q_H^2}{16} \frac{e^{4\beta(H)\ell_H} - 1}{2\beta(H)} \right]. \tag{67}$$

Then

$$\boxed{\mathcal{D}_H(\delta_H) \geq 2 \left(\beta(H) - \frac{1}{2} - \delta_H \right) \log T_H + \mathcal{L}_H^{\mathrm{loc}}.} \tag{68}$$

If CPM holds with exponent δ_H, then by definition

$$\mathcal{D}_H(\delta_H) \leq 0. \tag{69}$$

Consequently

$$\boxed{\beta(H) - \frac{1}{2} \leq \delta_H - \frac{\mathcal{L}_H^{\mathrm{loc}}}{2 \log T_H}.} \tag{70}$$

Proof. Equation (74) gives

$$|\mathcal{P}_\alpha^\circ(e^u)|^2 \geq \frac{q_H^2}{16} e^{(2\beta(H)-1)u}.$$

Multiply by the Jacobian e^u and integrate over J_H to obtain (65). Taking logarithms and subtracting $(1 + 2\delta_H) \log T_H$ gives (68). Under CPM the definition (66) immediately gives (69); combining the two inequalities yields (70). □

Remark 20.2 (Repair of StrandLoop P03 S16). A P03 completion arm correctly observed that the displayed lower bound for $\mathcal{D}_H$ together with a separately displayed prime-norm bound does not imply $\mathcal{D}_H \leq 0$ unless the defect is tied to that norm. Equation (66) is the missing definition. With it, CPM $\Rightarrow \mathcal{D}_H \leq 0$ is tautological rather than a new analytic gate.

Corollary 20.3 (Admissibility controls the local penalty from below)**.** *For an admissible sequence, condition (A3) implies*

$$\mathcal{L}_{H_n}^{\mathrm{loc}} \geq -2\varepsilon_n \log T_n - O(1). \tag{71}$$

Consequently, if CPM holds with $\delta_n \to 0$ and (73) *holds on the captured interval, then*

$$\boxed{\beta_n - \frac{1}{2} \leq \delta_n + \varepsilon_n + o(1).} \tag{72}$$

Thus the defect ledger recovers exactly the exponent separation used in Theorem 22.1; no upper $o(\log T_n)$ bound for $\mathcal{L}_{H_n}^{\mathrm{loc}}$ is required.

Proof. The admissibility constant is

$$c_n = \frac{q_{H_n}^2}{4} \frac{e^{4\beta_n \ell_{H_n}} - 1}{2\beta_n},$$

whereas the factor inside (67) is $c_n/4$. Condition (A3), $c_n^{1/2} \geq T_n^{-\varepsilon_n}$, therefore gives

$$\mathcal{L}_{H_n}^{\mathrm{loc}} = \log(c_n/4) \geq -2\varepsilon_n \log T_n - \log 4,$$

which is (71). Insert this lower bound into (70). □

21 Selection interface and forced-prime inheritance

Proposition 21.1 (Prime inherits a captured spectral lower bound)**.** *On a capture interval J_H from Theorem 15.3,*

$$|\mathcal{P}_\alpha^\circ(e^u)| \geq \frac{q_H}{2} e^{(\beta(H)-1/2)u} - |\mathcal{R}_\alpha^{\mathrm{can}}(e^u)|.$$

Thus whenever

$$|\mathcal{R}_\alpha^{\mathrm{can}}(e^u)| \leq \frac{q_H}{4} e^{(\beta(H)-1/2)u} \qquad (u \in J_H), \tag{73}$$

we have

$$|\mathcal{P}_\alpha^\circ(e^u)| \geq \frac{q_H}{4} e^{(\beta(H)-1/2)u} \qquad (u \in J_H). \tag{74}$$

Proof. Use (10), the reverse triangle inequality, and Theorem 15.3. □

Remark 21.2. This is not an independent contradiction; it is the pointwise form of the same prime–zero identity. Its value is diagnostic: any CPM proof on the selected interval must genuinely conflict with the forced record-edge scale, rather than rely on unrelated average behavior.

Let

$$\mathcal{B}_H := \{u : |G_H(u)| \geq q_H\}, \qquad A_H := \mathcal{B}_H \cap [U_H^-, U_H^+].$$

Remark 21.3 (Average-to-interval firewall). A bound

$$\frac{1}{\mathrm{meas}(A_H)} \int_{A_H} |\mathcal{P}_\alpha^\circ(e^u)|^2 \, du \leq e^{o(H^2)}$$

does not by itself imply the existence of a fixed-length interval $J_H \subset A_H$ with the same scale. The set A_H may have positive measure but empty ℓ-interior. Any averaging-based selection theorem must therefore control

$$A_{H,\ell}^\circ := \{u : [u-\ell, u+\ell] \subset A_H\}$$

or an equivalent thickness parameter.

Research Gate 21.4 (SI: common selection interface)**.** *If Gates 16.7, 17.4, and 19.1 cannot be established uniformly, prove a selection theorem producing one subsequence on which all of the following coexist:*

(SI1) a record-persistent Gaussian window with $D_H > 0$;

(SI2) exact lower-block separation (56)*;*

(SI3) a Bohr superlevel interval of controlled length and thickness;

(SI4) subpolynomial capture-quality loss;

(SI5) the heat-flow CPM bound (64) *on that same interval.*

22 Conditional exclusion

Theorem 22.1 (Conditional exclusion from canonical coupling)**.** *Assume* ¬RH *and suppose there exists an admissible Gaussian capture sequence satisfying Gate 19.1. If*

$$\liminf_{n\to\infty}\bigl(\beta_n - \tfrac{1}{2} - \varepsilon_n - \delta_n\bigr) > 0, \tag{75}$$

then that off-line forcing sequence is impossible.

Proof. By Proposition 18.2,

$$\|\mathcal{S}_\alpha^{\mathrm{G}}\|_{L^2(I_n)} \geq T_n^{\beta_n-\varepsilon_n}.$$

By (10), Gate 19.1, and (59),

$$\|\mathcal{S}_\alpha^{\mathrm{G}}\|_{L^2(I_n)} \leq T_n^{1/2+\delta_n} + O_\alpha(1),$$

contradicting (75) for large n. Equivalently, after the pointwise remainder has been absorbed, Proposition 20.1 gives the same contradiction as a positive lower bound for $\mathcal{D}_{H_n}(\delta_n)$ against the CPM identity $\mathcal{D}_{H_n}(\delta_n) \leq 0$. □

23 Why the second moment remains the first battlefield

The current contradiction already has the required exponent separation

$$T^{2\beta} \quad \text{versus} \quad T^{1+o(1)}, \qquad \beta > 1/2.$$

No fourth-moment or higher-moment hypothesis is used.

24 War Room targets

The record-edge branch remains organized around the same five gates as v1.4. Version 1.6 preserves the parallel intrinsic-source program and adds a defect–compensator audit layer, but neither addition renames nor dilutes the five record-edge gates.

R1. Gaussian persistence (GP). Under ¬RH, find record plateaus for which the edge was born far enough below the cutoff that the Gaussian high-tail advantage exceeds edge attenuation and all amplitude losses.

R2. Lower-block separation (LB). Control η_H and C_H/q_H using the exact quantity

$$\frac{\log(4C_H/q_H)}{\eta_H H^2},$$

and verify the cross-window inequality (56).

R3. Quantitative Bohr relative density (QB). Bound the maximum gap L_H between useful edge recurrences, not merely one simultaneous return, strongly enough to fit inside the surviving two-sided window.

R4. Heat-flow centered-prime mean square (CPM). Prove or kill (64) on the selected windows, using either the pointwise F representation or the unconditional tempered representation $\widetilde{\mathcal{P}_\alpha^\circ} = k_\alpha * \widetilde{F}$. Track the exact defect (66) rather than suppressing interval-width and amplitude losses.

R5. Selection interface (SI). If R1–R4 fail uniformly, select a common subsequence satisfying all four simultaneously and with enough interval thickness to support the second-moment contradiction.

Parallel intrinsic-source program (not a sixth gate). The source branch now has two closed structural steps and three open closure tasks:

S1. Positive source bridge — closed. Use Proposition 6.3 to pass exactly between $\widetilde{F}$ and M_ψ.

S2. Source injectivity/invariant — closed. Use Theorems 7.1 and 7.5; do not replace uniqueness by a false claim of stable global backward heat flow.

S3. Arithmetic separator — open. Construct Σ satisfying Placeholder 9.1.

S4. Zero–source bridge — open. Prove Placeholder 9.3 with multiplicity-safe explicit-formula bookkeeping.

S5. Separator kill test — active. Attempt the counterexample program in Gate 9.2 before promoting any candidate Σ.

Defect–compensator closure program (audit layer, not a sixth gate). The v1.6 structural program has five ordered tasks:

D1. Quotient/dual geometry — closed. Use Theorem 10.2; exact noncompensation is never substituted for positive distance from the closed range.

D2. Euler negative control — closed and numerically calibrated. Use Theorem 11.1 together with Section 13.1; the three-run control now records raw precision-floor, precision-retreat/coefficient-blow-up, and regularization-bias pseudo-gap mechanisms.

D3. Regime-I observer — closed at $a = 0$. Use $r = \widetilde{F} \in L^2([0, \infty))$ from Proposition 12.1. No positive exponential weight is installed without an additional theorem.

D4. Regime-I exact compensator and closure — open. Complete Gates 12.4 and 12.5 before asking for a numerical bore.

D5. Internal wall survey — active. Run Gate 13.5 on the paper's own walls, with EULER-CI as the mandatory negative control and KANDy restricted to exploratory pattern discovery.

Kill discipline. Failure of GP means the Gaussian suppresses fresh record edges faster than the high tail can be separated. Failure of LB means the lower finite block does not clear before the high-tail clock expires. Failure of QB means phase recurrence is too slow. Failure of CPM means the prime object is capable of carrying the forced off-line energy. Failure of SI means the individually favorable events do not coexist. Any one of these is a legitimate route-kill; none is to be repaired by changing the meaning of the gate after the fact.

25 Conclusion

Version 1.6 preserves the complete v1.5 record-edge and intrinsic-source architecture while adding a third object of a different type: a defect–compensator closure audit layer. It is not itself an RH proof branch and does not rename any existing gate.

The record-edge branch remains

$$\boxed{\begin{aligned}\neg\mathrm{RH} &\longrightarrow \text{record persistence} \longrightarrow \text{Bohr capture}\\ &\longrightarrow \text{forced local prime energy} \longrightarrow \text{CPM incompatibility}\end{aligned}}$$

subject to the unchanged five gates GP/LB/QB/CPM/SI. Nothing in v1.5 declares those gates solved.

The new source branch begins with an exact arithmetic object:

$$\boxed{M_\psi = \sum_{n\geq 2} \frac{\Lambda(n)}{n}\delta_{\log n} = (\partial_v + 1)(\widetilde{F} + 1), \qquad M_\psi \in \mathcal{M}_{+,\mathrm{temp}}(\mathbb{R}).}$$

Its Gaussian trajectory

$$Q_\alpha = G_\alpha * M_\psi$$

uniquely determines M_ψ on $\mathcal{S}'$, even though the inverse heat map is unstable. At the same time, the abstract A6 theorem proves that a decomposition $Q = R - S$ cannot be recovered from observers factoring only through Q on a nontrivial diagonal gauge orbit. The resulting information split is therefore permanent:

$$\boxed{\begin{aligned}&\text{spectral bookkeeping decomposition can be lost}\\ &\qquad \text{while the arithmetic source remains identifiable.}\end{aligned}}$$

The surviving intrinsic object is

$$\boxed{\mathfrak{U}_\alpha(Q) = \{\widehat{M_\psi}\}}$$

for every $\alpha > 0$. Generic positivity, however, is not enough. The load-bearing open problem is now the arithmetic separator:

$$\boxed{\begin{aligned}\Sigma(\widehat{M_\psi}) \quad &\text{must encode an unconditional prime-source constraint}\\ &\text{that a generic positive source need not satisfy.}\end{aligned}}$$

The unconditional RH closure remains exactly where it belongs: in Placeholders 9.1 and 9.3. Until those are proved, no proof of RH is claimed.

The v1.6 closure layer adds a different question beneath the existing walls. Its first certified infinite-dimensional test is negative:

$$\overline{\mathrm{span}\{(p^{s+1}-1)^{-1} : p \in \mathbb{P}\}}^{C(I)} = C(I), \qquad \ker B^* = \{0\}.$$

Thus the prime-ray Euler observer has no continuous bore in the uniform topology. The first live lower-block arena is instead

$$Y_0 = L^2([0,\infty)), \qquad r = \widetilde{F} \in Y_0,$$

with the exact selected-zero defect and the genuine compensator map still to be derived. The decisive v1.6 binary gate is therefore

$$\boxed{b_{\rho_0} \overset{?}{\in} \overline{B_0(C_0)}^{L^2([0,\infty))}.}$$

A negative answer must be certified by a continuous annihilator; a positive answer is a clean closure kill for this topology.

The numerical program now has a precise role. It first reproduces the Euler dense-range negative control, then surveys the paper's own walls and the Regime-I candidate through enrichment trajectories J_N, conditioning, and dual-witness stability. KANDy may locate a stable phase boundary or a candidate annihilator direction, but only an infinite-dimensional closure theorem can promote that signal.

The operational rule for subsequent work is therefore balanced. The record-edge five-gate program remains alive; the intrinsic-source separator program remains alive; neither is allowed to borrow an unproved conclusion from the other. Negative results, counterexamples, observer-kernel failures, and source-space restrictions are to be frozen as part of the theorem ledger rather than rewritten after the fact.

A Function-space and Fourier audit

This appendix fixes the conventions required by the source-injectivity claims.

Tempered distributions. $\mathcal{S}(\mathbb{R})$ denotes the Schwartz space and $\mathcal{S}'(\mathbb{R})$ its dual. A positive distribution is a positive Radon measure; a positive Radon measure with polynomial mass growth is tempered [7, 8]. Proposition 6.2 therefore places the actual M_ψ in the domain used by Theorem 7.1 without an RH assumption.

Fourier normalization. The paper uses the convention for which

$$\widehat{G_\alpha}(\xi) = e^{-\alpha\xi^2}$$

for $G_\alpha(v) = (4\pi\alpha)^{-1/2}e^{-v^2/(4\alpha)}$. A different 2π convention changes only the positive constant multiplying $\alpha\xi^2$ and none of the injectivity or positivity arguments.

Localized reciprocal lemma. If $a \in C^\infty(\mathbb{R})$ is nowhere zero and $aT = 0$ in $\mathcal{D}'$, then $T = 0$: for every compactly supported χ, the function χ/a is a valid test function and

$$\langle T, \chi\rangle = \langle aT, \chi/a\rangle = 0.$$

This lemma proves injectivity of multiplication by the Gaussian multiplier while avoiding the false claim that its global reciprocal is a tempered multiplier.

Positive-definite distributions. The Bochner theorem for functions and its Schwartz-distribution extension identify positive measures with positive-definite Fourier transforms [6, 7, 9]. In v1.6 this theorem is used only to characterize the generic positive-source cone. It is not itself an RH discriminator.

B Gauge quotient and observer algebra

Let

$$\Delta := \{(T,T) : T \in \mathscr{T}\}.$$

Then $\pi(R,S) = R - S$ is constant on cosets of Δ, and every Q-factored observer descends to the quotient $\mathscr{X}/\Delta$. Theorem 8.2 is therefore a quotient-identifiability statement. It does not say that the localized spectral term is zero; it says that the chosen observer algebra cannot distinguish its value along a gauge orbit.

The audit distinction can be summarized as

$$\boxed{\mathcal{O}(x) = \mathcal{O}(y) \;\not\Longrightarrow\; x = y \quad \text{unless injectivity is separately proved.}}$$

By contrast, Theorem 7.1 supplies exactly such an injectivity proof for the Gaussian map on the arithmetic source space.

C Research-instrument provenance and compiler contract

The August 13, 2026 StrandLooper runs are recorded as research instrumentation, not as external mathematical authority. Their purpose was to generate, attack, and compress candidate structures; every promoted theorem in the body has been rewritten as an ordinary mathematical statement with an ordinary proof.

Semantic-type failure discovered. A Round-3 compressed arm preserved the symbol G_α while losing the fact that it was the Gaussian heat kernel; a downstream adversary then replaced it by an unrelated finite-group signed kernel. That countermodel was rejected. The permanent compiler contract is

$$\boxed{\begin{aligned}&\text{SYMBOL} + \text{TYPE} + \text{DOMAIN}\\ &\qquad + \text{DEFINING IDENTITIES} + \text{NONNEGOTIABLE PROPERTIES.}\end{aligned}}$$

A symbol surviving compression is not semantic preservation.

Correlated-arm firewall. Agreement among multiple reasoning arms does not count as independent evidence if the arms share a compiler representation that has lost the same hypothesis. The preferred architecture is

$$\boxed{\begin{aligned}&\text{source} \to \{\text{compiler A}, \text{compiler B}, \text{full-context control}\}\\ &\qquad \to \text{arms} \to \text{cross-representation adversary.}\end{aligned}}$$

Discovery value and certification status are recorded separately.

Execution-isolation failure discovered. The final-round prompt requested five separate source strands, but the exported execution metadata showed one actual engine-level source strand. Model-generated internal labels "F1"–"F5" were therefore not treated as proof of execution isolation. Consequently, the final-round next-movement forecast is retained as mathematical exploration but not scored as a pristine blind prediction.[4]

[4] This is an instrumentation result, not a mathematical defect in Theorems 7.1 or 7.5, which stand on their explicit proofs.

Euler-CI calibration milestone. The numerical negative-control program has now passed its intended falsification test. The same operator whose closure gap is proved to be zero can exhibit (i) a finite-precision residual floor after effective-rank saturation, (ii) a residual retreat by many orders of magnitude when precision is increased, accompanied by explosive compensator coefficients, and (iii) a regularization-induced positive plateau. Consequently the paper now has an internal empirical standard for distinguishing a candidate structural gap from three common pseudo-gap mechanisms. This milestone does not move any RH gate by itself; it licenses the calibrated harness to interrogate BLC_0 only after the exact Regime-I compensator map is mathematically frozen.

D Closure ledger

Item	Status	Required before any RH claim
Canonical contour identity	Proved/retained	No reopening absent a detected algebraic error.
Record-edge GP/LB/QB/CPM/SI	Open gates	All conditions needed for the record-edge contradiction must coexist exactly as stated.
Positive source bridge	Proved	Proposition 6.3.
Gaussian source injectivity	Proved	Theorem 7.1.
Gauge non-identifiability	Proved structurally	Theorem 8.2; do not attempt component recovery through the same quotient.
Intrinsic invariant $\mathfrak{U}_\alpha(Q)$	Proved	Theorem 7.5.
Defect–compensator quotient theorem	Proved	Theorem 10.2; use closed-range distance, not algebraic nonmembership.
Prime-ray Euler $C(I)$ closure	Proved negative control	Theorem 11.1; dense range and trivial dual annihilator in this topology.
Regime-I L^2 observer	Proved at $a = 0$	Proposition 12.1; $r = \widetilde{F}$ is square-integrable while $A = 1 + r$ is not.
Regime-I exact compensator map	**OPEN**	Gate 12.4.
Regime-I closure/density	**OPEN**	Gate 12.5.
Internal wall data survey	Calibrated instrumentation	Gate 13.5; EULER-CI calibration is complete and the harness is ready for a definition-complete Regime-I operator.
Arithmetic separator Σ	**OPEN**	Prove all five requirements in Placeholder 9.1.
Zero–source incompatibility	**OPEN**	Prove Placeholder 9.3.
Counterexample/kill search	Active	Kill any separator imitated by a generic positive source.
Unconditional RH closure	**Open placeholder**	Only after the two open separator/bridge items are proved may Placeholder 9.4 be promoted.

Source-note inventory

Version 1.6 retains v1.2–v1.5 unchanged as historical predecessors and records the August 14, 2026 defect–

compensator continuation. The v1.5 gauge theorem, positive-source bridge, Gaussian injectivity, and α-independent source invariant remain unchanged. The new promoted mathematics is the quotient/dual bore criterion, the prime-ray Euler $C(I)$ density theorem used as a negative control, and the unweighted Regime-I membership $\widetilde{F} \in L^2([0,\infty))$. The Euler theorem has now also been calibrated numerically in three regimes: raw double-precision minimax, 50-decimal-digit precision retreat, and Tikhonov regularization. These experiments establish instrumentation signatures only, not new analytic theorems. The exact Regime-I zero defect, compensator map, and closure decision remain open and are not manufactured from schematic zero terms. The wall-survey/KANDy layer is research instrumentation only: finite-dimensional residuals and model agreement are never cited as proof, and the historical GP/LB/QB/CPM/SI gate definitions are preserved.

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